Charge-symmetry-breaking nucleon form factors

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Abstract A quantitative understanding of charge-symmetry breaking is an increasingly important ingredient for the extraction of the nucleon's strange vector form factors. We review the theoretical understanding of the charge-symmetry-breaking form factors, both for single nucleons and for ^{4}He.

1 Introduction

The investigation of strangeness contributions to static properties of the nucleon is particularly interesting as it gives unambiguous access to low-energy manifestations of virtual or sea quark effects. Different strangeness currents of the form $\bar{s}\Gamma s$ test the strangeness component of different nucleon observables, such as mass ($\Gamma = 1$), spin ($\Gamma = \gamma_\mu \gamma_5$), or magnetic moment ($\Gamma = \gamma_\mu$). Here we are concerned with the magnetic moment only, or, more generally, with the nucleon form factors of the vector current.

The Standard Model provides access to two different flavor combinations of the three light quark contributions to the electric (G_E) and magnetic (G_M) form factors due to the electromagnetic and the weak vector currents,

$$G_{E/M}^{\gamma,p} = \frac{2}{3} G_{E/M}^u - \frac{1}{3}\left(G_{E/M}^d + G_{E/M}^s\right) , \tag{1}$$

$$G_{E/M}^{Z,p} = \left(1 - \frac{8}{3}\sin^2\theta_W\right) G_{E/M}^u - \left(1 - \frac{4}{3}\sin^2\theta_W\right)\left(G_{E/M}^d + G_{E/M}^s\right) . \tag{2}$$

In order to obtain a full flavor decomposition of the vector current, one therefore has to invoke isospin (or charge) symmetry in the form

$$G_{E/M}^{u,n} = G_{E/M}^{d,p} , \quad G_{E/M}^{d,n} = G_{E/M}^{u,p} , \tag{3}$$

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and use the neutron electromagnetic form factors as the third input. These relations are at the heart of the extensive experimental program to extract the *weak* form factors of the proton $G_{E/M}^{Z,p}$ from parity-violating electron scattering [1,2,3,4,5,6,7,8,9,10]. If one relaxes this assumption and allows for charge-symmetry breaking, however, the relation between weak vector form factors, electromagnetic form factors of proton and neutron, and strangeness is complicated by an additional term,

$$G_{E/M}^{Z,p} = \left(1 - 4\sin^2\theta_W\right)G_{E/M}^{\gamma,p} - G_{E/M}^{\gamma,n} - G_{E/M}^s - G_{E/M}^{u,d} \,, \tag{4}$$

where $G_{E/M}^{u,d} = 2/3(G_{E/M}^{d,p} - G_{E/M}^{u,n}) - 1/3(G_{E/M}^{u,p} - G_{E/M}^{d,n})$. In other words, the charge-symmetry-violating form factors $G_{E/M}^{u,d}$ generate "pseudo-strangeness", and in order to reliably extract strangeness effects, the former have to be calculated from theory. This is becoming a necessity in particular in view of the increasingly tighter bounds on strangeness deduced from experiment [1,2,3,4,5,6,7,8,9,10].

2 Theory of charge-symmetry-breaking form factors

The isospin-breaking form factors $G_{E/M}^{u,d}$ have been addressed in various models of the strong interactions, in particular in constituent quark [11,12] or light-cone meson–baryon models [13] (see also Ref. [14] for a comparative review). These are afflicted by several problems: first, as in general with model calculations, it is extremely difficult to quantify the inherent uncertainties; second, the symmetries of the Standard Model may be violated. E.g., quark models [11,12] predict $G_{E/M}^{u,d}(t=0)=0$ for the charge-symmetry breaking magnetic moment, which is only due to a specific symmetry of the quark model wave function employed, not, as we shall see below, due to a symmetry of the Standard Model, and may consequently lead to an underestimation of isospin-breaking effects in particular at small t.

Chiral perturbation theory (ChPT) [15,16], on the other hand, is ideally suited for an analysis of isospin violation. It is tailor-made to analyze the dependence of low-energy observables on quark masses, in particular on the light quark mass difference $m_u - m_d$, and the consistent inclusion of electromagnetic effects is also well-understood [17]. As the isospin-violating form factors can be calculated in SU(2) ChPT, they are not affected by convergence problems to the extent the strangeness form factors are (see Ref. [18] for a review on the latter).

Particular emphasis will be put on the analysis of the leading moments of the isospin-violating form factors, magnetic moment as well as electric and magnetic radius terms,

$$G_E^{u,d}(t) = \rho_E^{u,d}t + \mathcal{O}(t^2)\,, \qquad G_M^{u,d}(t) = \kappa^{u,d} + \rho_M^{u,d}t + \mathcal{O}(t^2)\,. \tag{5}$$

The two radius terms are unaffected by low-energy constants up to leading ($\rho_E^{u,d}$) and next-to-leading ($\rho_M^{u,d}$) order and can be expressed entirely in terms of the neutron-to-proton mass difference $\Delta m = m_n - m_p$ [19], with the result [20]

$$\rho_E^{u,d} = \frac{5\pi C}{6M_\pi m_N}\,, \quad \rho_M^{u,d} = \frac{2C}{3M_\pi^2}\left\{1 - \frac{7\pi}{4}\frac{M_\pi}{m_N}\right\}\,, \quad C = \frac{g_A^2 m_N \Delta m}{16\pi^2 F_\pi^2}\,. \tag{6}$$

Note that these expressions for the radii are entirely non-analytic in the quark masses. Their simplicity is remarkable: the pion mass difference $M_{\pi^+}^2 - M_{\pi^0}^2$ cannot play a role as it only breaks charge *independence*, not charge symmetry; furthermore, up to $\mathcal{O}(p^4)$ for $G_E^{u,d}$ and
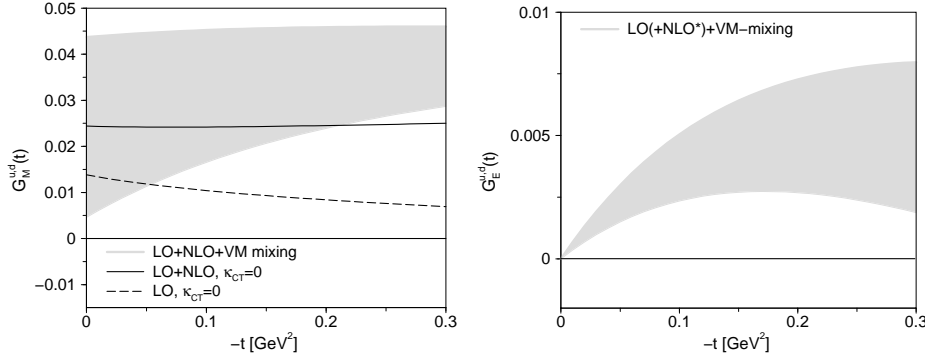
Fig. 1 Charge-symmetry-breaking magnetic (left) and electric (right) form factors. The grey bands denote the combined theoretical uncertainty due to various input parameters. Figures taken from Ref. [20].

$\mathcal{O}(p^5)$ for $G_M^{u,d}$, no photon loops contribute, nor are there any two-loop effects. In fact, ChPT is more predictive here than for the usual electromagnetic form factors of the nucleon (see e.g. Ref. [21]), precisely because polynomial counterterm contributions must be suppressed by isospin-breaking factors $m_u - m_d$ or e^2, and hence by two orders in the chiral expansion.

In order to complete the chiral representation, we have to estimate the combination of low-energy constants entering $\kappa^{u,d}$. This is done by invoking the principle of resonance saturation: low-energy constants in effective theories incorporate the effects of heavier states not included in the theory as explicit degrees of freedom. In our case, the relevant contributions are provided by vector mesons including $\rho - \omega$ mixing [20]:

$$G_E^{u,d}(t)\Big|_{\text{mix}} = \frac{\Theta_{\rho\omega}\,t}{M_V(M_V^2 - t)^2}\left[\left(1 + \frac{\kappa_\omega M_V^2}{4m_N^2}\right)g_\omega F_\rho - \left(1 + \frac{\kappa_\rho M_V^2}{4m_N^2}\right)g_\rho F_\omega\right],$$

$$G_M^{u,d}(t)\Big|_{\text{mix}} = \frac{\Theta_{\rho\omega}}{M_V(M_V^2 - t)^2}\left[\left(t + \kappa_\omega M_V^2\right)g_\omega F_\rho - \left(t + \kappa_\rho M_V^2\right)g_\rho F_\omega\right]. \tag{7}$$

The necessary couplings can be extracted from experimental data within certain errors. Such an inclusion of phenomenological vector-meson contributions has been shown to cure the main deficits of a chiral one-loop representation of the usual nucleon electromagnetic form factors [21], and is expected to work even better here due to the stronger suppression of even higher mass states in the mixing amplitudes. Still, the uncertainties in particular in the vector-meson–nucleon coupling constants $g_{\rho/\omega}$, $\kappa_{\rho/\omega}$ [22] limit the precision of the prediction for the isospin-violating form factors; see Ref. [20] for a detailed discussion.

Although, strictly speaking, the chiral representation of the form factors to $\mathcal{O}(p^4)$ ($G_E^{u,d}$) and $\mathcal{O}(p^5)$ ($G_M^{u,d}$) only requires an estimate for the low-energy constant entering the isospin-violating magnetic moment, the representation Eq. (7) also allows to assess higher-order counterterms contributing to the radii. Numerically one finds that, although formally subleading, the large vector-meson couplings tend to overwhelm the loop contributions Eq. (6) in the radii, which scale with the (small) nucleon mass difference. We therefore include the full mixing amplitudes in the final predictions.

The total results for the charge-symmetry-breaking form factors are shown in Fig. 1. The error bands combine an estimate of chiral corrections at higher order with the above-mentioned uncertainties in the input coupling constants for the resonance contributions. Although these combined uncertainties are sizeable, several conclusions can still be drawn: the

Table 1 Comparison of selected experimental measurements of strange form factors from SAMPLE [3], A4 [5], and HAPPEX [6] to the results of Ref. [20] for the isospin-violating form factors.

experiment	electric/magnetic	G^s	$G^{u,d}$
SAMPLE	G_M	$0.37 \pm 0.20 \pm 0.26 \pm 0.07$	$0.02\dots 0.05$
A4	$G_E + 0.106\,G_M$	0.071 ± 0.036	$0.004\dots 0.010$
HAPPEX	$G_E + 0.080\,G_M$	$0.030 \pm 0.025 \pm 0.006 \pm 0.012$	$0.004\dots 0.009$

effects of isospin breaking remain at the percent level; the t-dependence of the form factors is rather moderate in the low-energy region. We note that the symmetries of the Standard Model do *not* dictate $\kappa^{u,d}$ to vanish, indeed we find $G_M^{u,d}(0) \neq 0$.

The validity of these results, and in particular of the prediction for $\kappa^{u,d} = 0.025 \pm 0.020$ [20], has been criticized as an "extreme estimate" in Ref. [23] and too large in comparison to Ref. [12]; in particular the input on g_ω, κ_ω [22] has been questioned. This criticism is unwarranted for the following reasons. The central value for $\kappa^{u,d}$ is due to pion-loop contributions at a scale M_ρ, where changing the scale by a factor of 2 leads to a shift of 0.008 only. Completely scale-independent is the next-to-leading order correction in $\kappa^{u,d}$ (which is $\mathscr{O}(M_\pi)$, hence non-analytic in the quark masses), which can be read off as the difference between the full and the dashed curve in the magnetic form factor in Fig. 1, and which contributes roughly 40% of the central value. The potentially controversial vector meson contributions only lead to the uncertainty range of ± 0.020. Furthermore, the large anomalous ωN coupling found in Ref. [22] leads to the *lower* edge of the band in $G_M^{u,d}$, Fig. 1, hence to a sizeable cancellation with the pion-loop terms; reducing these couplings makes the total result larger, not smaller. Quite to the contrary, the quark model results for $G_M^{u,d}$ [12] are too small at low t by wrongly enforcing $G_M^{u,d}(t=0)$ to vanish.

Table 1 compares the specific linear combinations of $G_E^{u,d}$ and $G_M^{u,d}$ at $t \approx -0.1\,\mathrm{GeV}^2$ with the experimentally extracted values for strangeness form factors. We find that the charge-symmetry-breaking shifts are still smaller than other experimental uncertainties, but should be kept in mind for precision determinations of strange matrix elements. As another illustration, in the latest combined analysis of forward and backward asymmetries at A4 [9], the following values for the strange form factors were extracted at $t = -0.22\,\mathrm{GeV}^2$:

$$G_E^s = 0.050 \pm 0.038 \pm 0.019 \ , \quad G_M^s = -0.14 \pm 0.11 \pm 0.11 \ . \tag{8}$$

Isospin breaking shifts the central values according to $0.050 \rightarrow 0.045$ for G_E^s and $-0.14 \rightarrow -0.18$ for G_M^s, with negligible additional errors due to the theoretical uncertainties of ± 0.002 and ± 0.01, respectively. Again, these shifts are within the given error bars, but already of a comparable size.

3 Isospin mixing in Helium-4

Parity-violating electron scattering on ^{4}He gives clean access to the strange *electric* form factor G_E^s, as the $J^\pi = 0^+$ target does not allow for magnetic or axial vector transitions. However, in addition to effects of charge-symmetry breaking in the electric form factor as discussed in the previous section, an $I = 1$ admixture in the ^{4}He wave function yields a contribution to the measured asymmetry A_{PV} [24],

$$A_{PV} = -\frac{G_\mu t}{4\pi\alpha\sqrt{2}}\left\{4\sin^2\theta_W + \Gamma\right\}, \quad \Gamma = -2\frac{F^{(1)}}{F^{(0)}} - \frac{2G_E^\gamma - G_E^s}{(G_E^p + G_E^n)/2}, \tag{9}$$

where $F^{(0/1)}$ are the nuclear form factors corresponding to isoscalar/isovector charge operators, and $G_E^\gamma = (G_E^{u,p} - G_E^{d,n} - G_E^{d,p} + G_E^{u,n})/4$ is a different isospin-breaking linear combination of single-nucleon form factors. The measured asymmetry $A_{PV} = \left[+6.40 \pm 0.23_{stat} \pm 0.12_{syst}\right] \times 10^{-6}$ at $t = -0.077\,\mathrm{GeV}^2$ [8] leads to $\Gamma = 0.010 \pm 0.038$. Single-nucleon isospin violation contributes 0.008 ± 0.003 to Γ, while isospin mixing in the ^{4}He wave function amounts to ≈ 0.003, leaving a mere strangeness contribution of $G_E^s = -0.001 \pm 0.016$.

4 Conclusions

We have performed an analysis of the charge-symmetry-breaking nucleon form factors, using a combination of chiral perturbation theory and resonance saturation that relies as far as possible on the symmetries of the Standard Model and experimental data. Although we predict the isospin-breaking magnetic moment $G_M^{u,d}(0)$ to be different from zero (in contrast to certain model predictions), both electric and magnetic form factors are small, and their momentum-transfer dependence moderate. The contributions of isospin violation to parity-violating asymmetries are as yet smaller than some of the experimental uncertainties in extracting strange form factors, but clearly, charge-symmetry-breaking effects become an essential ingredient for precision extractions of strangeness matrix elements.

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